SAMPLE HINTS AND SOLUTIONS

INSTITUTE NAME & LOGO

JEE-MAIN EXAM YEAR		
Time : 60 Min	**Maths : Full Portion Paper**	**Marks : 100**
Hints and Solutions		

51) Ans: **2)** $\sqrt{3}$

Sol: $\tan^{-1} x + \cot^{-1} x + \cot^{-1} x = \frac{2\pi}{3}$

$\Rightarrow \cot^{-1} x = \frac{2\pi}{3} - \frac{\pi}{2} = \frac{\pi}{6} \Rightarrow x = \sqrt{3}$

52) Ans: **2)** 9

Sol: $10^n + 3\,(4^{n+2}) + 5$

Put $n = 2$; $10^2 + 3 \times 4^4 + 5$ =100+768+5=873

$\therefore$ This is divisible by 9

53) Ans: **4)** (4,3,-2)

Sol: a parallel to line OA is $\hat{i} - 2\hat{j} - k$.

b parallel to line OB is $3\hat{i} - 2\hat{j} + 3k$.

$$\therefore a \times b = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 1 & -2 & -1 \\ 3 & -2 & 3 \end{vmatrix} = -8\hat{i} - 6\hat{j} + 4k$$

Required DR's are -8,-6,4 or 4,3,-2.

54) Ans: **3)** $\frac{8}{21}$

Sol: Let, $I = \int_0^{\pi/2} \sqrt{\cos\theta}\, \sin^3\theta \, d\theta$

Putting $t = \cos\theta \quad \Rightarrow dt = -\sin\theta\, d\theta$, then

$I = -\int_1^0 t^{1/2}(1 - t^2)dt = \int_0^1 (t^{1/2} - t^{5/2})dt$

$I = \left[\frac{2}{3}t^{3/2} - \frac{2}{7}t^{7/2}\right]_0^1 = \frac{8}{21}$

55) Ans: **1)** Two plus two is four

56) Ans: **2)** 10

Sol: Here, first term $a = 10$, last term $l = 50$ and sum $S = 300$

$\therefore S = \frac{n}{2}(a + l) \Rightarrow 300 = \frac{n}{2}(10 + 50) \Rightarrow n = 10$

57) Ans: **1)** $\frac{3}{\frac{1}{30} + \frac{1}{25} + \frac{1}{50}}$ km/hr

Sol: Average speed

$= \frac{120 + 120 + 120}{\frac{120}{30} + \frac{120}{25} + \frac{120}{50}} = \frac{3}{\frac{1}{30} + \frac{1}{25} + \frac{1}{50}}$ km / hr

58) Ans: **1)** $y^2 - 10x - 6y + 14 = 0$

Sol: Let, the centre be (h, k), then radius = h (from given)

Also, $CC_1 = R_1 + R_2$

$\therefore \sqrt{(h-3)^2 + (k-3)^2} = h + \sqrt{9 + 9 - 14}$

$\Rightarrow (h-3)^2 + (k-3)^2 = h^2 + 4 + 4h$

$\Rightarrow k^2 - 10h - 6k + 14 = 0 \Rightarrow y^2 - 10x - 6y + 14 = 0$

59) Ans: **1)** g = 2, f = 3, c is any number.

Sol: The lines are parallel, if $h^2 = ab, af^2 = bg^2$.

or $\frac{a}{h} = \frac{h}{b} = \frac{g}{f} \Rightarrow 4f^2 = 9g^2 \Rightarrow f = \frac{3}{2}g \Rightarrow g = 2, \text{if } f = 3$

Now, $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

$\Rightarrow 4 \times 9 \times c + 2 \times 3 \times 2 \times 6 - 4(3)^2 - 9(2)^2 - c(6)^2 = 0$

i.e. c is any number.

60) Ans: **1)** 2^{2n}

Sol: Suppose the original set consists of $(2n+1)$ elements, then subsets of this set containing more than n elements means subsets containing $(n+1)$ elements, $(n+2)$ elements, ... $(2n+1)$ elements.

$\therefore$ Required number of subsets

$= {}^{2n+1}C_{n+1} + {}^{2n+1}C_{n+2} + ... + {}^{2n+1}C_{2n} + {}^{2n+1}C_{2n+1}$

$= {}^{2n+1}C_n + {}^{2n+1}C_{n-1} + ... + {}^{2n+1}C_1 + {}^{2n+1}C_0$

$= {}^{2n+1}C_0 + {}^{2n+1}C_1 + {}^{2n+1}C_2 + ... + {}^{2n+1}C_{n-1} + {}^{2n+1}C_n$

$= \frac{1}{2}\left[(1+1)^{2n+1}\right] = \frac{1}{2}[2^{2n+1}] = 2^{2n}$

61) Ans: **3)** $\frac{1}{4\sqrt{2}}$

Sol: We have

$\lim_{x\to 0} \frac{\sqrt{2} - \sqrt{1 + \cos x}}{\sin^2 x}$

$= \lim_{x\to 0} \frac{2 - (1 + \cos x)}{\sin^2 x} \times \frac{1}{\sqrt{2} + \sqrt{1 + \cos x}}$

$= \lim_{x\to 0} \frac{1 - \cos x}{(1 + \cos x)(1 - \cos x)} \times \lim_{x\to 0} \frac{1}{\sqrt{2} + \sqrt{1 + \cos x}}$

$= \lim_{x\to 0} \frac{1}{(1 + \cos x)} \times \frac{1}{2\sqrt{2}} = \frac{1}{4\sqrt{2}}$

62) Ans: **2)** $\frac{1}{12}$

Sol: Required probability $= \frac{2+1}{36} = \frac{1}{12}$

63) Ans: **4)** isosceles triangle.

Sol: Given that $\overrightarrow{AB}=\overrightarrow{BC}$. Thus, it is an isosceles triangle.

64) Ans: **1)** $\sqrt{3}x+y\pm 10=0$

Sol: Let p be the length of perpendicular from origin on the line, then its equation in normal form is $x\cos 30^o + y\sin 30^o = p$ or $\sqrt{3}x+y=2p$

It meets the coordinate axes at $A\left(\frac{2p}{\sqrt{3}},0\right)$ and $B(0,2p)$.

$\therefore$ Area of $\Delta OAB=\frac{1}{2}\left(\frac{2p}{\sqrt{3}}\right)2p=\frac{2p^2}{\sqrt{3}}$

By hypothesis $\frac{2p^2}{\sqrt{3}}=\frac{50}{\sqrt{3}} \quad \Rightarrow p=\pm 5$.

The lines are $\sqrt{3}x+y\pm 10=0$

65) Ans: **1)** $y=\pm\sqrt{(\log t)^2+C}$

Sol: The given equation is

$$t=1+(ty)\left(\frac{dy}{dt}\right)+\frac{(ty)^2}{2!}\left(\frac{dy}{dt}\right)^2+\ldots\infty \qquad \Rightarrow t=e^{ty\left(\frac{dy}{dt}\right)}$$

$$\Rightarrow \log t = ty\frac{dy}{dt} \;\Rightarrow y\,dy=\frac{\log t}{t}dt$$

On integrating both sides, we get $\frac{y^2}{2}=\frac{(\log t)^2}{2}+k$

$$\Rightarrow y=\pm\sqrt{(\log t)^2+2k} \;\Rightarrow y=\pm\sqrt{(\log t)^2+C}$$

66) Ans: **1)** $\log\left|e^x+e^{-x}\right|+C$

Sol: $\int\frac{e^{2x}-1}{e^{2x}+1}dx=\int\frac{(e^x.e^x-1)}{(e^x.e^x+1)}dx$

$$=\int\frac{e^x\left(e^x-\frac{1}{e^x}\right)}{e^x\left(e^x+\frac{1}{e^x}\right)}dx=\int\frac{(e^x-e^{-x})}{(e^x+e^{-x})}dx$$

Let $e^x+e^{-x}=t \Rightarrow e^x-e^{-x}=\frac{dt}{dx}$

$$\Rightarrow dx=\frac{dt}{e^x-e^{-x}}$$

$$\int\frac{e^{2x}-1}{e^{2x}+1}dx=\int\frac{(e^x-e^{-x})}{t}\,\frac{dt}{e^x-e^{-x}}=\int\frac{1}{t}dt=\log|t|+c$$

$$=\log\left|e^x+e^{-x}\right|+C$$

67) Ans: **4)** axis.

Sol: Normal to parabola will be, $y=mx-2am-am^3$

For three values of m, three normal can be drawn on parabola $y^2=4ax$.

Thus, three feet of normals can be obtained, hence centroid of triangle lies on axis of parabola.

68) Ans: **2)** 1/4

Sol: $\sin\frac{\pi}{10}\sin\frac{3\pi}{10}=\sin 18^\circ.\sin 54^\circ$

$$=\sin 18^\circ.\cos 36^\circ \Rightarrow =\frac{\sqrt{5}-1}{4}.\frac{\sqrt{5}+1}{4}=\frac{1}{4}$$

69) Ans: **1)** 101^{50}

Sol: We know, $101^{50}=(100+1)^{50}$

$$=100^{50}+50.100^{49}+\frac{50.49}{2.1}100^{48}+\ldots \quad \ldots..(i)$$

and $99^{50}=(100-1)^{50}$

$$=100^{50}-50.100^{49}+\frac{50.49}{2.1}100^{48}-\ldots \quad \ldots..(ii)$$

By subtracting (ii) from (i), we get,

$$101^{50}-90^{50}=100^{50}+2\frac{50.49.48}{1.2.3}100^{47}>100^{50}$$

$\therefore 101^{50}>100^{50}+99^{50}$

70) Ans: **4)** $\theta=\frac{n\pi}{7}+\frac{\pi}{14}$

Sol: From given, $\tan 5\theta=\tan\left(\frac{\pi}{2}-2\theta\right)$

$$\Rightarrow 5\theta=n\pi+\frac{\pi}{2}-2\theta \Rightarrow 7\theta=n\pi+\frac{\pi}{2} \Rightarrow \theta=\frac{n\pi}{7}+\frac{\pi}{14}$$

71) Ans: **15** Sol: For $m=5$, $\sum_{i=0}^{5}\binom{10}{i}\binom{20}{5-i}$

$$=\binom{10}{0}\binom{20}{5}+\binom{10}{1}\binom{20}{4}+\ldots..+\binom{10}{5}\binom{20}{0}$$

for m=10, $\sum_{i=0}^{10}\binom{10}{i}\binom{20}{10-i}$

$$=\binom{10}{0}\binom{20}{10}+\binom{10}{1}\binom{20}{9}+\binom{10}{2}\binom{20}{8}+..+\binom{10}{10}\binom{20}{0},$$

for m = 15, $\sum_{i=0}^{15}\binom{10}{i}\binom{20}{15-i}$

$$=\binom{10}{0}\binom{20}{15}+\binom{10}{1}\binom{20}{14}+\binom{10}{2}\binom{20}{13}+..+\binom{10}{10}\binom{20}{5}$$

and for m = 20, $\sum_{i=0}^{20}\binom{10}{i}\binom{20}{20-1}\binom{20}{20-i}$

$$=\binom{10}{0}\binom{20}{20}+\binom{10}{1}\binom{20}{19}+\ldots.+\binom{10}{10}\binom{20}{10}$$

Clearly, the sum is maximum for m = 15.

Note that ${}^{10}C_r$ is maximum for r = 5 and ${}^{20}C_r$ is maximum for r = 10.

Also, the single term ${}^{10}C_5\times{}^{20}C_{10}$ (in case m = 15) is greater than the sum

${}^{10}C_0\,{}^{20}C_{10}+{}^{10}C_1\,{}^{20}C_9+{}^{10}C_2\,{}^{20}C_8+\ldots..$

$+{}^{10}C_8\,{}^{20}C_2+{}^{10}C_9\,{}^{20}C_1+{}^{10}C_{10}\,{}^{20}C_0$ (in case m = 10).

Also, the sum in case m = 10 is same as that in case m = 20.

72) Ans: **1** Sol: Let $z_1 = x_1 + iy_1, z_2 = x_2 + iy_2$

$$\left|\frac{z_1 - 3z_2}{3 - z_1\bar{z}_2}\right| = 1$$

$$\Rightarrow \frac{(x_1 - 3x_2)^2 + (y_1 - 3y_2)^2}{(3 - x_1x_2 - y_1y_2)^2 + (-x_1y_2 + x_2y_1)^2} = 1$$

$$\Rightarrow x_1^2 + 9x_2^2 - 6x_1x_2 + y_1^2 + 9y_2^2 - 6y_1y_2$$

$$= 9 + x_1^2x_2^2 + y_1^2y_2^2 - 6x_1x_2 - 6y_1y_2 + 2x_1x_2y_1y_2 + x_1^2y_1^2 + x_2^2y_2^2 - 2x_1x_2y_1y_2$$

$$\Rightarrow (x_1^2y_1^2) + 9(x_2^2 + y_2^2) - 6(x_1x_2 + y_1y_2)$$

$$= 9 + (x_1^2 + y_1^2)(x_2^2 + y_2^2) - 6(x_1x_2 + y_1y_2)$$

$$= |z_1|^2 + 9|z_2|^2 = 9 + |z_1|^2|z_2|^2$$

$$\Rightarrow |z_2^2|(9 - |z_1|^2) = 9 - |z_1|^2 \Rightarrow |z_2|^2 = 1 \Rightarrow |z_2| = 1$$

73) Ans: **1** Sol: As, $A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -3 \\ 2 & 1 & 0 \end{bmatrix}$

$$\Rightarrow B = \text{adj}(A) = \begin{bmatrix} 3 & 1 & 1 \\ -6 & -2 & 3 \\ -4 & -3 & 2 \end{bmatrix}$$

$$\Rightarrow \text{adj}(B) = \begin{bmatrix} 5 & -5 & 5 \\ 0 & 10 & -15 \\ 10 & 5 & 0 \end{bmatrix}$$

$$\Rightarrow |\text{adj}(B)| = \begin{vmatrix} 5 & -5 & 5 \\ 0 & 10 & -15 \\ 10 & 5 & 0 \end{vmatrix} = 625$$

Given that, $C = 5A$

$$\Rightarrow |C| = 5^3|A| = 125\begin{vmatrix} 1 & -1 & 1 \\ 0 & 2 & -3 \\ 2 & 1 & 0 \end{vmatrix} = 625$$

$$\therefore \frac{|\text{adj}(B)|}{|C|} = \frac{625}{625} = 1$$

74) Ans: **-1** Sol: $(x^2 + x - 2)(x^2 + x - 3) = 12$

$\Rightarrow (y - 2)(y - 3) = 12$ where $y = x^2 + x$

$\Rightarrow y^2 - 5y - 6 = 0$

$\Rightarrow y = 6, -1 \Rightarrow x^2 + x = 6, x^2 + x = -1$

$x^2 + x = 6 \Rightarrow x = -3, 2$ are real roots and

$x^2 + x + 1 = 0$ has complex roots $\frac{-1 \pm \sqrt{3}i}{2}$

Therefore, sum of non-real

roots $= \left(\frac{-1 + \sqrt{3}\,i}{2}\right) + \left(\frac{-1 - \sqrt{3}\,i}{2}\right) = -1$

75) Ans: **22** Sol:

$n(M' \cap P' \cap C') = n(M \cup P \cup C)' = n(U) - n(M \cup P \cup C)$

$= 175 - (S_1 - S_2 + S_3)$

$= 175 - \{(100 + 70 + 46) - (30 + 28 + 23) + 18\}$

$= 175 - \{(216 - 81 + 18\} = 175 - 153 = 22$